Probing the anisotropic velocity of light in a gravitational field: another test of general relativity

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Abstract

A corollary of general relativity that the average velocity of light between two points in a gravitational field is anisotropic has been overlooked. It is shown that this anisotropy can be probed by an experiment which constitutes another test of general relativity.

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Although the bending of light and the zero velocity of light which tries to leave a gravitationally collapsing body are indications that the propagation of light in a gravitational field is anisotropic, that anisotropy has been barely mentioned in the case of the coordinate velocity of light only [1], [2]. It has been overlooked that the average velocities of light toward and away from a gravitational center between two points of different gravitational potential are not equal to c and not the same. As we shall see bellow the propagation of light in non-inertial reference frames (accelerating or at rest in a gravitational field) is anisotropic (direction-dependent). This can most clearly be demonstrated by revisiting the issue of propagation of light in the Einstein elevator experiment [3] and studying the propagation of light rays parallel to the elevator's acceleration (in addition to the horizontal ray originally considered by Einstein).

Consider first an elevator accelerating with an acceleration $a = |\mathbf{a}|$ which represents a non-inertial (accelerating) reference frame N^a (Figure 1). Three light rays are emitted simultaneously in the elevator (in N^a) from points D, A, and C toward point B. Let I be an inertial reference frame instantaneously at rest with respect to N^a (i.e. the comoving frame) at the moment the light rays are emitted. The emission of the rays is therefore simultaneous in N^a as well as in I. At the next moment an observer in I sees that the three light rays arrive simultaneously not at point B, but at B' since for the time $t = h/c$ the light rays travel toward B the elevator moves at a distance $\delta = at^2/2 = ah^2/2c^2$. As the simultaneous arrival of the three rays at point B' as viewed in I is an absolute event being a *point* event, it follows that the rays arrive simultaneously at B' as seen from N^a as well. Since for the *same* coordinate time $t = h/c$ in N^a the three light rays travel different distances $DB' \approx h$, $AB' = r + \delta$, and $CB' = r - \delta$ before arriving simultaneously at point B' an observer in the elevator concludes that the *average* downward velocity $c^a_\downarrow$ of the light ray propagating from A to B' is slightly greater than c

$$c^a_\downarrow = \frac{h+\delta}{t} \approx c\left(1 + \frac{ah}{2c^2}\right). \tag{1}$$

The average upward velocity $c^a_\uparrow$ of the light ray propagating from C to B' is slightly smaller than c

$$c^a_\uparrow = \frac{h-\delta}{t} \approx c\left(1 - \frac{ah}{2c^2}\right). \tag{2}$$

Therefore the average velocities of light rays traveling parallel and anti-parallel to the elevator's acceleration are different from c and from each other. An observer in N^a will conclude that the three light rays arrive at B' (not at B) due to the anisotropy in the propagation of light in the elevator which in turn is caused by its accelerated motion.

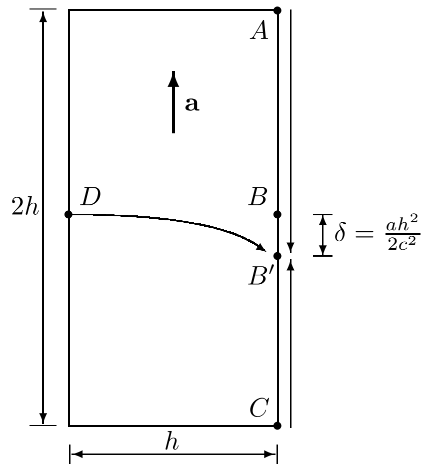

Figure 1. Three light rays propagate in an accelerating elevator. After having been emitted simultaneously from points A, C, and D the rays meet at B'. The ray propagating from D toward B, but arriving at B', represents the original thought experiment considered by Einstein. The light rays emitted from A and C are introduced in order to determine the expression for the average anisotropic velocity of light in an accelerating frame of reference. It takes the same time $t = h/c$ for the rays to travel the distances $DB' \approx h$, $AB' = h+\delta$, and $CB' = h-\delta$. Therefore the average velocity of the downward ray from A to B' is $c^a_\downarrow = (h+\delta)/t \approx c(1+ah/2c^2)$; the average velocity of the upward ray from C to B' is $c^a_\uparrow = (h-\delta)/t \approx c(1-ah/2c^2)$.

As seen from (1) and (2) the average anisotropic velocity of light in N^a involves accelerations and distances for which $ah/2c^2 < 1$. This restriction is always satisfied since it is weaker than the one imposed by the principle of equivalence which requires that only small regions in a gravitational field where the field is uniform are considered [4] (see also the paragraph immediately after equation (4)).

Consider now an elevator (i.e. a non-inertial reference frame N^g) at rest in the Earth's gravitational field. The elevator will appear accelerating upward (with an acceleration $g = |\mathbf{g}|$) with respect to a reference frame I which is at rest with respect to N^g, but starts to fall in the gravitational field at the moment the light rays are emitted. During the time the light rays emitted from the points A, C, and D travel toward B the elevator will appear to move with respect to I at a distance $\delta = gt^2/2 = gh^2/2c^2$ and for this reason the three light rays will meet not at B but at B' situated bellow B at a distance δ. Therefore an observer in N^g also finds that the propagation of light is anisotropic in the elevator. The average velocity of the light ray traveling from A to B' along $\mathbf{g}$ is

$$c^g_\downarrow = \frac{h+\delta}{t} \approx c\left(1 + \frac{gh}{2c^2}\right). \tag{3}$$

The average velocity of the light ray propagating from C to B' is slightly smaller than c [5]:

$$c^g_\uparrow = \frac{h-\delta}{t} \approx c\left(1 - \frac{gh}{2c^2}\right). \tag{4}$$

One may deduce from (4) that $gh/2c^2 < 1$ but this is an apparent restriction. The average velocity of light between two points separated by a greater distance in a strong gravitational field is determined in terms of the explicit difference of the gravitational potentials of the two points [6].

The average anisotropic velocities (3) and (4) describe the propagation of light between *two* points in a gravitational field, separated by a distance h, since gh is the difference of the gravitational potential of the two points. It should be specifically noted that those velocities can be obtained from the expression for the velocity of light in a gravitational field derived by Einstein in 1911 $c' = c\left(1 + \Delta\Phi/c^2\right)$ [7], where $\Delta\Phi$ is the difference of the gravitational potential of the two points. This fact demonstrates that the 1911 Einstein velocity of light, which led him to a wrong value of the deflection of light by the Sun, has been prematurely abandoned. In his 1916 paper [8] he obtained the correct deflection angle by using the *coordinate* velocity of light $c' = c(1 - 2GM/Rc^2)$. However, as this velocity depends on the gravitational potential of one point only it does not describe the propagation of

light between two points in a gravitational field; it is the 1911 expression for the velocity of light that provides the correct description of this case.

One can also get the average velocities (3) and (4) directly from (1) and (2) by using the equivalence principle and substituting $a = g$ in (1) and (2) [9].

The velocities (3) and (4) demonstrate that there exists a directional dependence in the propagation of light between two points in a gravitational field. This anisotropy in the propagation of light is an overlooked corollary of general relativity. Therefore an experiment for testing that anisotropy constitutes another test of general relativity.

The purpose of this paper is to propose an experiment to test the average anisotropic velocity of light in a gravitational field. The experiment to be described bellow is based on another experiment which was proposed by Stolakis [10] in 1986 with the intention to measure the one-way velocity of light. It turned out that this could not be done but it was pointed out that the experiment he proposed might be used for testing a possible anisotropy of spacetime [11]. The experiment can be described in the following way. Consider again the Einstein elevator at rest in the Earth's gravitational field (Figure 2). At point B a light beam is split into two beams 1 and 2 which propagate vertically (with respect to the Earth's surface). Ray 1 travels the distance h upward from B to A in a medium of index of refraction n; at A it is reflected by a mirror and its return path toward B is in vacuum ($n = 1$). Ray 2 also travels the same distance h in a medium of refractive index n but downward from B to C; its return path after being reflected by a mirror at C is in vacuum too. Upon their arrival at point B rays 1 and 2 interfere. If the average velocity of light is anisotropic the interference pattern produced by vertically propagating rays will differ from the interference pattern of two horizontally traveling rays.

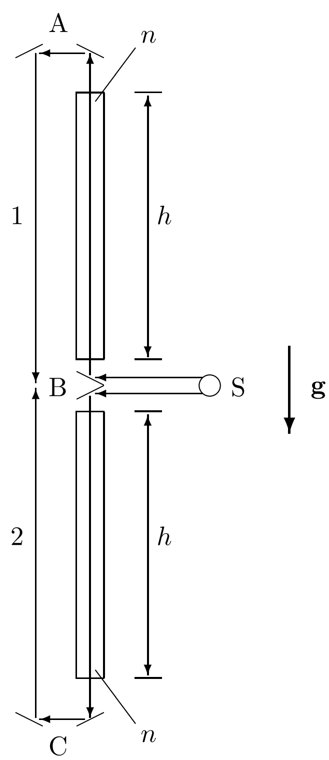

Figure 2. An experiment for measuring the average anisotropic velocity of light in a gravitational field of strength $\mathbf{g}$. At point B a light ray coming from a source S is split into two rays 1 and 2 which propagate in a medium of refractive index n toward points A and C, respectively. After having traveled back to B through a vacuum the light rays interfere at B. Upon its arrival at B light ray 1 (after having been reflected by a mirror at A) is delayed by $\Delta t = (gh^2/c^3)(n-1)$ with respect to ray 2 coming from C. The reason for the delay is that the velocity of ray 1 on its way from B to A is doubly slowed down - by the medium and the gravitational filed.

Taking into account (4) and (3) the upward average velocity of ray 1 from B to A in the medium is

$$c_n^{\uparrow} = \frac{c_{\uparrow}^{g}}{n} = \frac{c}{n}\left(1 - \frac{gh}{2c^2}\right).$$

while the downward average velocity of ray 2 from B to C also in the medium is

$$c_n^{\downarrow} = \frac{c_{\downarrow}^{g}}{n} = \frac{c}{n}\left(1 + \frac{gh}{2c^2}\right).$$

Then the time for which ray 1 goes to A through the medium and returns to B through the vacuum is

$$t^1 = \frac{h}{c_n^{\uparrow}} + \frac{h}{c_{\downarrow}^{g}} \approx \frac{hn}{c}\left(1 + \frac{gh}{2c^2}\right) + \frac{h}{c}\left(1 - \frac{gh}{2c^2}\right). \tag{5}$$

The time for which ray 2 travels through the medium to C and returns to B in the vacuum is

$$t^2 = \frac{h}{c_n^{\downarrow}} + \frac{h}{c_{\uparrow}^{g}} \approx \frac{hn}{c}\left(1 - \frac{gh}{2c^2}\right) + \frac{h}{c}\left(1 + \frac{gh}{2c^2}\right). \tag{6}$$

The difference between the two time intervals t^1 and t^2 is

$$\Delta t = t^1 - t^2 = \frac{gh^2}{c^3}(n-1). \tag{7}$$

It is seen from (5) and (6) that $\Delta t \neq 0$ (for $n > 1$) only if the average velocity of light is anisotropic, i.e. only if it is different from c. If the return paths of rays 1 and 2 are in a medium of index of refraction n' then obviously (7) becomes

$$\Delta t = \frac{gh^2}{c^3}(n - n').$$

The delay Δt is best visualized by using a spacetime diagram as shown in Figure 3.

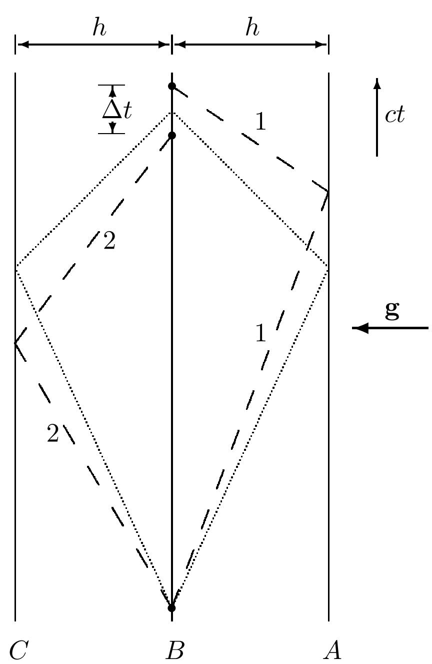

Figure 3. Spacetime diagram of the propagation of light rays 1 and 2 as shown in Figure 2. If the average velocity of light between two points in a gravitational field is anisotropic, the worldlines of the rays will be represented by the dash lines; if the velocity of light is c the rays will be represented by the dotted lines. The vertical worldlines depict points A, B, and C.

Performed as described here the experiment cannot detect a change in the interference pattern of two light rays one of which is delayed by Δt that is $\sim 10^{-23}s$ for $h = 10m$. This tiny delay is equivalent to a shift of the wavelength of one of the rays with respect to the other's wavelength by $10^{-5}nm$ which cannot produce an observable change in the interference pattern. If, however, the two rays are made to pass multiple times through the medium toward points A and C, respectively and through the vacuum back to point B, the delay Δt will accumulate and the effect may become detectable [12].

References

[1] L. Schiff, *Am. J. Phys.* **28**, 340 (1960).

[2] H. Ohanian and R. Ruffini, *Gravitation and Spacetime*, 2nd ed., (W. W. Norton, New York, London, 1994), p. 197.

[3] A. Einstein, L. Infeld, *The Evolution of Physics*, (Simon & Schuster, New York, 1966).

[4] The equivalence principle can be applied only to regions of dimension h in a gravitational field which are small enough (such that $gh/2c^2 \ll 1$) in order to ensure that the field is uniform there.

[5] The average velocities of light (3) and (4) can be directly obtained by noting that the propagation of light is affected by the Earth's gravity. At B' the instantaneous velocity of the ray emitted at C will be slowed down by the Earth's gravity: $c_{\uparrow}^{B'} = c - gt \approx c(1 - gh/c^2)$. As the initial velocity of the ray is c its average velocity from C to B' is $c_{\uparrow}^{g} = c(1 - gh/2c^2)$. The ray emitted at A is falling in the Earth's gravitational filed while traveling downward and will arrive at B' with the instantaneous velocity $c_{\downarrow}^{B'} = c + gt \approx c(1 + gh/c^2)$ and the ray's average velocity from A to B' will be $c_{\downarrow}^{g} = c(1 + gh/2c^2)$.

[6] In the case of strong gravitational fields and a large distance between two points A and B the average velocity of light propagating against the field from B toward A is $c_{BA}^{\uparrow} = c\left(1 + GM/c^2R_A - GM/c^2R_B\right)$. For a light ray traveling along the field from A to B the average light velocity is $c_{AB}^{\downarrow} = c\left(1 - GM/c^2R_A + GM/c^2R_B\right)$ (see V. Petkov, gr-qc/9909081).

[7] A. Einstein, *Ann. Phys.* **35**, 898 (1911).

[8] A. Einstein, *Ann. Phys.* **49**, 769 (1916).

[9] Strictly speaking, the equivalence principle requires that **a** be substituted with **-g**. To do this one should obtain the vector form of the average light velocity in N^a by taking A, B, B' and C to lie on a line making an angle with **a**: $c^a = c\left(1 - \mathbf{a}\cdot\mathbf{h}/2c^2\right)$. Then substituting **a** with **-g** yields $c^g = c\left(1 + \mathbf{g}\cdot\mathbf{h}/2c^2\right)$.

[10] G. Stolakis, *Brit. J. Phil. Sci.* **37**, 229 (1986).

[11] V. Petkov, *Brit. J. Phil. Sci.* **40**, 69 (1989).

[12] For a further development of the experiment proposed here see S. Manly and E. Page, "Experimental feasibility of measuring the gravitational redshift of light using dispersion in optical fibers", gr-qc/0008004.